

## High Performance Computing

2<sup>nd</sup> appello - February 13, 2013

*The answers can be written in English or in Italian. Please, present the work in a legible and readable form. All the answers must be properly and clearly explained.*

### Question 1

A stream parallel computation  $\Sigma$  has the following characteristics:

1. input stream element = (*opcode*,  $x$ ), output stream element = ( $y$ ), where  $x$  and  $y$  are of integer type, and *opcode* = {0, 1, 2, 3} identifies one of four distinct operations to be applied to  $x$ ;
2.  $\Sigma$  is implemented by a *functional partitioning* scheme with 4 workers  $W_0, W_1, W_2, W_3$ , each one specialized for a distinct operation identified by *opcode*. The four operations are equi-probable;
3. processing elements have a scalar pipelined CPU, with clock cycle  $\tau$ , time slot equal to  $2\tau$ , parallel Execution Unit with 4-stage pipelined integer functional units, 32K on-demand primary data cache, on-chip secondary cache;
4. let  $u = 10^4 \tau$ ;
5. the interarrival time of  $\Sigma$  is equal to  $u$ ;
6.  $T_{setup} = u/10, T_{transm} = u/100$ , zero-copy communication and communication processors;
7. worker  $W_i$ , for  $i = 0, 1, 2$ , has its own integer internal state  $S_i$ , and is defined as follows:  
 $\forall$  input  $a$  : output  $b = F_i(a, S_i); S_i = G_i(a, S_i)$ ;
8. the mean processing times of  $F_0, G_0, F_1, G_1, F_2, G_2$  are respectively equal to:  $u, u, 4u, 8u, u, 2u$ ;
9. worker  $W_3$  encapsulates an integer array  $C[M]$ , with  $M = 10^3$ , and is defined as follows:

$\forall$  input  $a$  : output  $b =$  number of elements of  $C$  which are integer multiples of  $a$ .

For the evaluation of  $W_3$  processing time, assume that the event “an element of  $C$  is an integer multiple of  $a$ ” has negligible probability.

- a) Evaluate the service time and the relative efficiency of  $\Sigma$  and of each module belonging to  $\Sigma$  implementation.
- b) Transform possible bottlenecks in order to improve the service time of  $\Sigma$ .

### Question 2

- a) Explain the following sentence: “In the performance evaluation of a parallel program on a shared memory architecture, the values of interprocess communication parameters  $T_{setup}$  and  $T_{transm}$  depend, in general, *also* on the parallel program itself (structure and implementation)”.
- b) Explain under which conditions we can assume that, with acceptable approximation,  $T_{setup}$  and  $T_{transm}$  are independent of the parallel program.

### Question 3

With reference to a pipelined scalar CPU, explain in quantitative terms what is the impact of the Execution Unit parallelization on performance.

## Solution

*to be integrated with proper explanations*

### Question 1

a) The functional partitioning implementation of  $\Sigma$  consists of a distributor module IN, the four workers, and a collector module OUT.

The ideal service times of IN and OUT are respectively equal to  $T_{send}(2)$  and  $T_{send}(1)$ , in practice both are equal to  $T_{setup} = u/10 \ll T_A$ , thus they are not bottlenecks.

According to the multiple server theorem, the interarrival time to any worker is equal to  $4u$ .

The ideal service times of the first three workers are:

$$T_{0-id} = T_{F0} + T_{G0} = 2u$$

$$T_{1-id} = T_{F1} + T_{G1} = 12u$$

$$T_{2-id} = T_{F2} + T_{G2} = 3u$$

All the communications are fully overlapped to computation.

$W_0$  and  $W_2$  are not bottlenecks, while  $W_1$  is a bottleneck.

Let us evaluate the ideal service time of  $W_3$ , without considering the communication primitives (see above), under the assumptions about the CPU architecture. The pseudo-code is:

```

int C[M]; int a, b; int s = 0;
for (i = 0; i < M, i++)
    if ( C[i] % a ) = 0
        s++;

```

which is compiled and optimized as follows:

```

LOAD Rvtg, 0, Ra
LOAD RC, Ri, Rc, don't_deallocate
1. LOOP: MOD Rc, Ra, Rmod
2. INCR Ri
3. IF ≠ 0 Rmod, CONT
4. INCR Rc
5. CONT: IF < Ri, RM, LOOP, delayed_branch
6. LOAD RC, Ri, Rc, don't_deallocate

```

(Delayed branch could be applied to  $IF \neq 0$  using  $INCR Ri$ , but consequently the distance of the logical dependency induced by instruction 1 becomes 1).

We have to pay the effect of a branch (instruction 3), with probability  $\lambda = 1/6$ , and of a logical dependency induced by instruction 1 on 3, of distance  $k = 2$ , probability  $d_k = 1/6$ ,  $N_Q = 2$ ,  $L_{pipe-k} = 4$  and no long-latency instruction in the critical sequence ( $\Delta_2 = 0$ ). The dependency of 2 on 5 has no effect. Thus, the service time per instruction, without cache faults, is given by:

$$T = (1 + \lambda)t + \Delta_1 = (1 + \lambda)t + t d_k (N_Q + L_{pipe-k} + 1 - k) = 2t$$

The completion time without cache faults is:

$$T_{c0} = 6 M T = 12 M t = 24 M \tau$$

Array  $C$  has the property of *reuse*: after the first stream element application,  $C$  is permanently maintained in cache. Thus, the cache fault penalty is negligible, and the ideal service time of  $W_3$  is :

$$T_{3-id} = 24 M \tau = 2.4 u$$

$W_3$  is not a bottleneck.

In conclusion, only  $W_1$  is a bottleneck, and its steady-state interarrival time becomes  $12u$ . The steady-state interarrival time of  $W_0, W_2, W_3$  must be re-evaluated. The interdeparture time from IN becomes:

$$\frac{1}{4} T_{1-id} = 3u$$

Thus, the steady-state interarrival time to  $W_0, W_2, W_3$  are:

$$\frac{3u}{\frac{1}{4}} = 12u$$

According to the multiple clients theorem, the interarrival time to OUT, equal to the effective service time of  $\Sigma$ , is given by:

$$\frac{1}{T_W} = \frac{1}{T_{p0}} + \frac{1}{T_{p1}} + \frac{1}{T_{p2}} + \frac{1}{T_{p3}} = \frac{1}{T_{A-0}} + \frac{1}{T_{1-id}} + \frac{1}{T_{A-2}} + \frac{1}{T_{A-3}} = \frac{4}{12u}$$

Thus, we confirm that the effective service time is equal to the steady-state interarrival time of  $\Sigma$ :

$$T_W = 3u$$

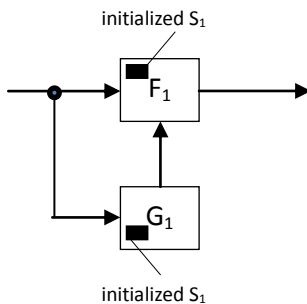
$$\varepsilon_W = \frac{T_{W-id}}{T_W} = \frac{T_A}{T_W} = \frac{u}{3u} = 0.33$$

For the component modules:

Module	ideal service time	effective service time	efficiency
IN	$\sim T_{setup} = 0.1 u$	$T_W = 3 u$	$\rho_{IN} = 0,033$
OUT	$\sim T_{setup} = 0.1 u$	$T_W = 3 u$	$\rho_{OUT} = 0,033$
$W_0$	$2 u$	$12 u$	$\rho_{W0} = 0,17$
$W_1$	$12 u$	$12 u$	$1$
$W_2$	$3 u$	$12 u$	$\rho_{W2} = 0,25$
$W_3$	$2.4 u$	$12 u$	$\rho_{W3} = 0,2$

b) Only the data-flow paradigm can be applied to parallelize the bottleneck module  $W_1$ , because it is a module with state and operates on elementary types.

AND-logic data-flow graph of  $W_1$



$F_1$  and  $G_1$  can be executed in parallel on  $a$ , and the new state produced of  $G_1$  is sent to  $F_1$  (this communication has negligible effect on the service time).

Thus, because  $F_1$  has to wait for the updated state, the effective service time of  $W_1$  is reduced to  $T_{G1} = 8u$ , and  $W_1$  remains a bottleneck. All the steady-state interarrival times of workers are now equal to  $8u$ .

For  $\Sigma$  we have:

$$T_W = 2 u$$

$$\varepsilon_W = 0.5$$

### Question 2

a) Parameters  $T_{setup}$  and  $T_{transm}$  are proportional to the under-load latency  $\Omega$  for memory access. Notably,  $T_{setup} \sim 5\Omega$  and  $T_{transm} \sim \Omega/\sigma$  for typical architectural characteristics.

The under-load latency is a function of the base latency  $\Omega_0$  and of parameters  $p$  and  $T_p$  (*insert their definition*) which depend on the parallel program structure (used parallel paradigms), on *parallelism degree*  $n$ , and on program *mapping*.

In general, in order to evaluate a parallel program, we need to apply an iterative procedure (determine  $n$  for a by-experience initial value of  $T_{setup}$  and  $T_{transm}$ ; then determine  $\Omega_0$ ,  $p$  and  $T_p$ , which depend also on  $n$ ; then determine new values of  $T_{setup}$  and  $T_{transm}$ ; then re-evaluate  $n$ , ...) until convergence. (*Explain clearly the relative dependencies among these parameters*)

b)  $T_{setup}$  and  $T_{transm}$  are approximately independent of the parallel program if two conditions hold:

- i) the impact of  $p$  and  $T_p$  on  $\Omega$  is limited in such a way that we can assume that  $\Omega \sim \Omega_0$ ;
- ii) because  $\Omega_0$  is a function of the average *distance* between nodes, this introduces another element of dependency on the parallel program mapping. Thus, the approximation is acceptable if the mapping and/or the network is not too sensible to the distance.

### Question 3

The EU parallelization is able to minimize the EU ideal service time (one time slot): this has an obvious positive effect on the ideal service time of the architecture and, most important, on the mean waiting time in queue,  $W(\rho)$ , of instructions delivered to the EU server. In fact, the delay  $\Delta$  incurred by IU for logical dependency is proportional to the EU response time:

$$R_Q = W(\rho) + L_s$$

On the other hand, the EU functional units are parallelized according to the pipeline paradigm, which has a negative effect on EU latency  $L_s$  compared to an ideal realization with latency equal to the service time (*explain why, in practice, the pipeline paradigm is the only parallelization forms for streams of arithmetic operations*). This is the reason for which the goal of the logical dependency optimizations is to mask the effects of the EU latency.