

## High Performance Computing

2<sup>nd</sup> appello - February 13, 2013

*The answers can be written in English or in Italian. Please, present the work in a legible and readable form.  
All the answers must be properly and clearly explained.*

### Question 1

A stream parallel computation  $\Sigma$  has the following characteristics:

1. input stream element =  $(opcode, x)$ , output stream element =  $(y)$ , where  $x$  and  $y$  are of integer type, and  $opcode = \{0, 1, 2, 3\}$  identifies one of four distinct operations to be applied to  $x$ ;
2.  $\Sigma$  is implemented by a *functional partitioning* scheme with 4 workers  $W_0, W_1, W_2, W_3$ , each one specialized for a distinct operation identified by *opcode*. The four operations are equi-probable;
3. processing elements have a scalar pipelined CPU, with clock cycle  $\tau$ , time slot equal to  $2\tau$ , parallel Execution Unit with 4-stage pipelined integer functional units, 32K on-demand primary data cache, on-chip secondary cache;
4. let  $u = 10^4 \tau$ ;
5. the interarrival time of  $\Sigma$  is equal to  $u$ ;
6.  $T_{\text{setup}} = u/10, T_{\text{transm}} = u/100$ , zero-copy communication and communication processors;
7. worker  $W_i$ , for  $i = 0, 1, 2$ , has its own integer internal state  $S_i$ , and is defined as follows:

$$\forall \text{ input } a : \text{ output } b = F_i(a, S_i); S_i = G_i(a, S_i);$$

8. the mean processing times of  $F_0, G_0, F_1, G_1, F_2, G_2$  are respectively equal to:  $u, u, 4u, 8u, u, 2u$ ;
9. worker  $W_3$  encapsulates an integer array  $C[M]$ , with  $M = 10^3$ , and is defined as follows:

$$\forall \text{ input } a : \text{ output } b = \text{number of elements of } C \text{ which are integer multiples of } a.$$

For the evaluation of  $W_3$  processing time, assume that the event “an element of  $C$  is an integer multiple of  $a$ ” has negligible probability.

- a) Evaluate the service time and the relative efficiency of  $\Sigma$  and of each module belonging to  $\Sigma$  implementation.
- b) Transform possible bottlenecks in order to improve the service time of  $\Sigma$ .

### Question 2

- a) Explain the following sentence: “In the performance evaluation of a parallel program on a shared memory architecture, the values of interprocess communication parameters  $T_{\text{setup}}$  and  $T_{\text{transm}}$  depend, in general, *also* on the parallel program itself (structure and implementation)”.
- b) Explain under which conditions we can assume that, with acceptable approximation,  $T_{\text{setup}}$  and  $T_{\text{transm}}$  are independent of the parallel program.

### Question 3

With reference to a pipelined scalar CPU, explain in quantitative terms what is the impact of the Execution Unit parallelization on performance.

## Solution

*to be integrated with proper explanations*

### Question 1

a) The functional partitioning implementation of  $\Sigma$  consists of a distributor module IN, the four workers, and a collector module OUT.

The ideal service times of IN and OUT are respectively equal to  $T_{send}(2)$  and  $T_{send}(1)$ , in practice both are equal to  $T_{setup} = u/10 \ll T_A$ , thus they are not bottlenecks.

According to the multiple server theorem, the interarrival time to any worker is equal to  $4u$ .

The ideal service times of the first three workers are:

$$\begin{aligned} T_{0-id} &= T_{F0} + T_{G0} = 2u \\ T_{1-id} &= T_{F1} + T_{G1} = 12u \\ T_{2-id} &= T_{F2} + T_{G2} = 3u \end{aligned}$$

All the communications are fully overlapped to computation.

$W_0$  and  $W_2$  are not bottlenecks, while  $W_1$  is a bottleneck.

Let us evaluate the ideal service time of  $W_3$ , without considering the communication primitives (see above), under the assumptions about the CPU architecture. The pseudo-code is:

```
int C[M]; int a, b; int s = 0;
for (i = 0; i < M, i++)
    if ( C[i] % a ) = 0
        s++;
```

which is compiled and optimized as follows:

```
LOAD  Rvtg, 0, Ra
LOAD  RC, Ri, Rc, don't_deallocate
1. LOOP:   MOD  Rc, Ra, Rmod
2.          INCR  Ri
3.          IF ≠ 0  Rmod, CONT
4.          INCR  Rc
5. CONT:    IF <  Ri, RM, LOOP, delayed_branch
6.          LOAD  RC, Ri, Rc, don't_deallocate
```

(Delayed branch could be applied to  $IF \neq 0$  using  $INCR\ Ri$ , but consequently the distance of the logical dependency induced by instruction 1 becomes 1).

We have to pay the effect of a branch (instruction 3), with probability  $\lambda = 1/6$ , and of a logical dependency induced by instruction 1 on 3, of distance  $k = 2$ , probability  $d_k = 1/6$ ,  $N_Q = 2$ ,  $L_{pipe-k} = 4$  and no long-latency instruction in the critical sequence ( $\Delta_2 = 0$ ). The dependency of 2 on 5 has no effect. Thus, the service time per instruction, without cache faults, is given by:

$$T = (1 + \lambda)t + \Delta_1 = (1 + \lambda)t + t d_k (N_Q + L_{pipe-k} + 1 - k) = 2t$$

The completion time without cache faults is:

$$T_{c0} = 6 M \quad T = 12 M \quad t = 24 M \quad \tau$$

Array  $C$  has the property of *reuse*: after the first stream element application,  $C$  is permanently maintained in cache. Thus, the cache fault penalty is negligible, and the ideal service time of  $W_3$  is :

$$T_{3-id} = 24 M \tau = 2.4 u$$

$W_3$  is not a bottleneck.

In conclusion, only  $W_1$  is a bottleneck, and its steady-state interarrival time becomes  $12u$ . The steady-state interarrival time of  $W_0, W_2, W_3$  must be re-evaluated. The interdeparture time from IN becomes:

$$\frac{1}{4} T_{1-id} = 3u$$

Thus, the steady-state interarrival time to  $W_0, W_2, W_3$  are:

$$\frac{3u}{\frac{1}{4}} = 12u$$

According to the multiple clients theorem, the interarrival time to OUT, equal to the effective service time of  $\Sigma$ , is given by:

$$\frac{1}{T_W} = \frac{1}{T_{p0}} + \frac{1}{T_{p1}} + \frac{1}{T_{p2}} + \frac{1}{T_{p3}} = \frac{1}{T_{A-0}} + \frac{1}{T_{1-id}} + \frac{1}{T_{A-2}} + \frac{1}{T_{A-3}} = \frac{4}{12 u}$$

Thus, we confirm that the effective service time is equal to the steady-state interarrival time of  $\Sigma$ :

$$T_W = 3 u$$

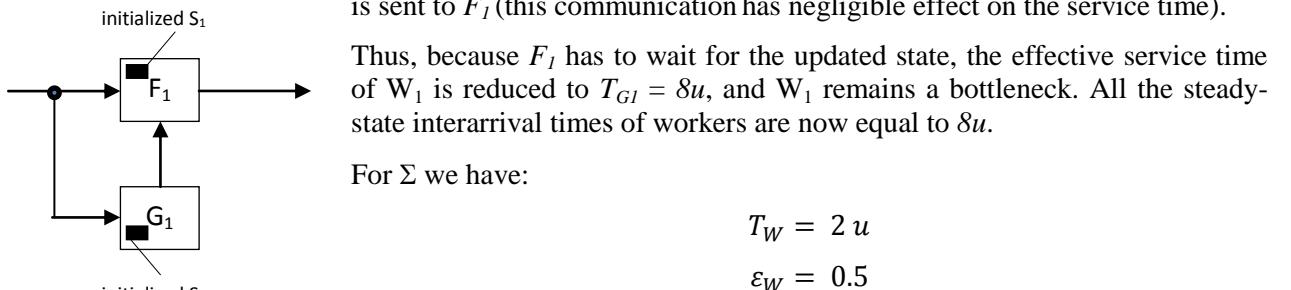
$$\varepsilon_W = \frac{T_{W-id}}{T_W} = \frac{T_A}{T_W} = \frac{u}{3 u} = 0.33$$

For the component modules:

Module	ideal service time	effective service time	efficiency
IN	$\sim T_{setup} = 0.1 u$	$T_W = 3 u$	$\rho_{IN} = 0,033$
OUT	$\sim T_{setup} = 0.1 u$	$T_W = 3 u$	$\rho_{OUT} = 0,033$
$W_0$	$2 u$	$12 u$	$\rho_{W0} = 0,17$
$W_1$	$12 u$	$12 u$	$1$
$W_2$	$3 u$	$12 u$	$\rho_{W2} = 0,25$
$W_3$	$2.4 u$	$12 u$	$\rho_{W0} = 0,2$

**b)** Only the data-flow paradigm can be applied to parallelize the bottleneck module  $W_1$ , because it is a module with state and operates on elementary types.

**AND-logic data-flow graph of  $W_3$**   $F_1$  and  $G_1$  can be executed in parallel on  $a$ , and the new state produced of  $G_1$  is sent to  $F_1$  (this communication has negligible effect on the service time).



### Question 2

**a)** Parameters  $T_{\text{setup}}$  and  $T_{\text{transm}}$  are proportional to the under-load latency  $\Omega$  for memory access. Notably,  $T_{\text{setup}} \sim 5\Omega$  and  $T_{\text{transm}} \sim \Omega/\sigma$  for typical architectural characteristics.

The under-load latency is a function of the base latency  $\Omega_0$  and of parameters  $p$  and  $T_p$  (insert their definition) which depend on the parallel program structure (uswed parallel paradigms), on *parallelism degree*  $n$ , and on program *mapping*.

In general, in order to evaluate a parallel program, we need to apply an iterative procedure (determine  $n$  for a by-experience initial value of  $T_{\text{setup}}$  and  $T_{\text{transm}}$ ; then determine  $\Omega_0$ ,  $p$  and  $T_p$ , which depend also on  $n$ ; then determine new values of  $T_{\text{setup}}$  and  $T_{\text{transm}}$ ; then re-evaluate  $n$ , ...) until convergence. (Explain clearly the relative dependencies among these parameters)

**b)**  $T_{\text{setup}}$  and  $T_{\text{transm}}$  are approximately independent of the parallel program if two conditions hold:

- i) the impact of  $p$  and  $T_p$  on  $\Omega$  is limited in such a way that we can assume that  $\Omega \sim \Omega_0$ ;
- ii) because  $\Omega_0$  is a function of the average *distance* between nodes, this introduces another element of dependency on the parallel program mapping. Thus, the approximation is acceptable if the mapping and/or the network is not too sensible to the distance.

### Question 3

The EU parallelization is able to minimize the EU ideal service time (one time slot): this has an obvious positive effect on the ideal service time of the architecture and, most important, on the mean waiting time in queue,  $W(\rho)$ , of instructions delivered to the EU server. In fact, the delay  $\Delta$  incurred by IU for logical dependency is proportional to the EU response time:

$$R_Q = W(\rho) + L_s$$

On the other hand, the EU functional units are parallelized according to the pipeline paradigm, which has a negative effect on EU latency  $L_s$  compared to an ideal realization with latency equal to the service time (explain why, in practice, the pipeline paradigm is the only parallelization forms for streams of arithmetic operations). This is the reason for which the goal of the logical dependency optimizations is to mask the effects of the EU latency.